“Recent Advances in Classical and Quantum Integrability”

Book of Abstracts

Leeds CaLISTA Workshop 2-6 June 2025

- **Simonetta Abenda - Tropical curves and KP solitons: a combinatorial approach**

  Abstract: In this talk I will present recent results obtained in collaboration with C. Fevola, T.O. Celik and Y. Mandelstham on a pure combinatorial approach to classification of KP soliton solutions on tropical curves. This talk is connected to the one proposed by Claudia Fevola.

- **Alexander Bobenko - Dimers and M-curves. Limit shapes from Riemann surfaces**

  Abstract: We develop a general approach to dimer models analogous to Krichever's scheme in the theory of integrable systems. This leads to dimer models on doubly periodic bipartite graphs with quasiperiodic positive weights. Dimer models with periodic weights and Harnack curves are recovered as a special case. This generalization from Harnack curves to general M-curves, which are in the focus of our approach, leads to transparent algebro-geometric structures. In particular, the Ronkin function and surface tension are expressed as integrals of meromorphic differentials on M-curves. Using variational descriptions, explicit representations for limit shapes are obtained in terms of Abelian integrals. Based on Schottky uniformization of Riemann surfaces, we compute the weights and dimer configurations. The computational results are in complete agreement with the theoretical predictions. The talk is based on joint works with N. Bobenko and Yu. Suris.

- **Alexander Bobenko - Solving the Bonnet problem. A hands-on adventure in 17 chapters**

  Public lecture

- **Irina Bobrova - How to detect non-commutative Painlevé property?**

  Abstract: Being one of the powerful method in the theory of integrable ordinary differential equations, the Painlevé property has led to discovery of new special functions known as the Painlevé transcendents. These special functions play a significant role in various branches of mathematics and mathematical physics. Since they present some classes of solutions to different scalar and matrix integrable PDEs, it is natural to study the classification of the Painlevé transcendents in a non-commutative setting. However, in this framework, the Painlevé property becomes more complicated.

  To address this issue, we propose alternative criteria that are extremely useful for identifying candidates which can be regarded as non-commutative analogs of the Painlevé transcendents. This talk is based on a series of papers in collaboration with Vladimir Sokolov.

- **Harry Braden - Affine Toda equations: applications and questions**

Abstract: Some recent applications of the affine Toda equations arising as reductions of Nahm's equations will be described. This leads to a number of open questions, variations of interesting problems previously studied.

- **Boris Bychkov - KP integrability in topological recursion**

  Abstract: Topological recursion of Chekhov–Eynard–Orantin is a remarkable universal recursive procedure that has been found in many enumerative geometry problems, from combinatorics of maps, to random matrices, Gromov-Witten invariants, Hurwitz numbers, Mirzakhani's hyperbolic volumes of moduli spaces, knot polynomials. A recursion needs an initial data: a spectral curve, and the recursion defines the sequence of invariants of that spectral curve. Kadomtsev–Petviashvili hierarchy is an integrable hierarchy of nonlinear PDEs. Except for many important properties, it quite often appears in the applications: a lot of functions from combinatorics, mathematical physics, theory of moduli spaces and Gromov–Witten theory are solutions to the KP hierarchy. In the talk I will define the KP integrability property for the topological recursion invariants and show that TR invariants are KP integrable if and only if the corresponding spectral curve is rational. If time permits I will discuss the construction of the KP tau function on the TR spectral curve of any genus which can be seen as a non-perturbative generalization of the Krichever's construction of the KP tau function on any algebraic curve. The talk is based on the series of joint works with A. Alexandrov, P. Dunin-Barkowski, M. Kazarian and S. Shadrin (https://arxiv.org/abs/2309.12176, https://arxiv.org/abs/2406.07391, https://arxiv.org/abs/2412.18592).

- **Debora Choińska - On solutions of integrable discretizations of Ernst-type equations**

  Abstract: Ernst-type equations are elegant reformulations of Einstein's vacuum equations of general relativity when the existence of two commuting Killing vector fields is assumed. Axisymmetric, stationary spacetimes such as rotating black holes and planar gravitational waves are examples of solutions of the Ernst-type equations.

- **Thibault Congy - New hydrodynamic reductions in soliton gas kinetic theory**

  Abstract: The soliton structure plays a fundamental role in many physical systems due to its fundamental feature: its shape remains unchanged after the collision with another soliton in the case of integrable dynamics. Such particle-like behaviour has been at the origin of a new mathematical object: the soliton gas, consisting of an incoherent collection of solitons for which phases and spectral parameters are randomly distributed. The dynamics of such a gas is governed by an integro-differential kinetic equation. To date, few analytically tractable reductions of this equation have been identified. In this work we propose a new, physically motivated ansatz that captures interactions between a soliton gas and a mean field. As a concrete example, we construct exact solutions for a multi-component soliton gas interacting with a mean flow, where the kinetic equation reduces to a diagonalisable quasilinear system. This work was conducted in collaboration with Gennady El (Northumbria University) and Mark Hoefer (University of Colorado, Boulder).

- **Marta Dell'Atti - Lagrangian multiform for finite dimensional systems: the open Toda lattice**

  Abstract: Lagrangian multiforms provide a variational framework to describe integrable hierarchies [1], and the case of Lagrangian 1-forms covers finite-dimensional integrable systems. We use the theory of Lie dialgebras for r-matrices introduced by Semenov-Tian-Shansky [2] to construct a general Lagrangian 1-form. Given a Lie dialgebra associated with a Lie algebra $\mathfrak{g}$ and a collection H_k, $k = 1, \ldots, N$, of invariant functions on $\mathfrak{g}^*$, we give a formula for a Lagrangian multiform describing the commuting flows for H_k on a coadjoint orbit in $\mathfrak{g}^*$. We focus on the open Toda chain by constructing two different Lie dialgebra structures on $(N+1)$, one with a non-skew-symmetric r-matrix and one with a skew-symmetric r-matrix. In both cases, we find a new set of canonical variables describing the chain for which we determine the relation with the Flaschka coordinates. The talk is based on a joint work with V. Caudrelier and A.A. Singh [3].

[1] S. Lobb, F. W. Nijhoff, *Lagrangian multiforms and multidimensional consistency*, J Phys A Math Theor 42(45), 454013 (2009).

[2] M. A. Semenov-Tian-Shansky, *Integrable Systems: the r-matrix Approach*, RIMS-1650, Kyoto University (2008).

[3] V. Caudrelier, M. Dell'Atti, A. A. Singh, *Lagrangian multiforms on coadjoint orbits for finite-dimensional integrable systems*, Lett Math Phys 114, 34 (2024).

- **Linden Disney-Hogg - Monopole spectral curves: integrability and computation**

  Abstract: BPS monopoles on Euclidean and hyperbolic space give rise to integrable systems which linearise on the Jacobian of an associated spectral curve. I will describe how symmetries and computer algebra tools allow us to simplify these and find new solutions.

- **Adam Doliwa - Non-commutative Hirota equation: geometry, symmetry, applications and new solutions**

  Abstract: The non-Abelian Hirota–Miwa system was proposed by Nimmo [18] as a generalization of Hirota's discrete Kadomtsev–Petviashvilii equation [4, 15, 16, 19] for non-commuting dependent variables, see also [17]. Its solutions generated by the Darboux transformation were given in [13, 18] in terms of the quasideterminants [11, 12], which provide an important technical and conceptual tool in the non-commutative linear algebra.

  In the first part of my talk I would like to recall some structural features of the non-commutative Hirota system starting from its geometric interpretation [6], which provides directly the four-dimensional consistency of the system, encoded in the – fundamental for projective geometry – Desargues' theorem. The symmetry of the Desargues configuration leads then to the A-type root lattice description of the Hirota equations and the corresponding affine Weyl group symmetry of the system. The splitting of the resulting tetrahedron map [7] in two pentagon maps, called in [5] the normalization map and the Veblen flip map, turns out to be important in the lattice statistical physics systems [1], where they are known as the spider move and the resplit move, respectively. These results allow for the quantum reduction, where the Weyl commutation relations are being imposed, which in the classical limit results in the corresponding Poisson map.

  The second part of my lecture is devoted to presentation of a new class of solutions of the non-commutative Hirota system, which originate from the rational approximation [3] and interpolation applications of the integrable systems [14], and the theory of multiple orthogonal polynomials [2]. These solutions, which generalize those found recently in [8, 9, 10], are also given in terms of quasideterminants, and are related to a new multidimensional,

non-autonomous and non-commutative integrable system, which generalizes the discrete Toda lattice equation.

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[2] A. I. Aptekarev, *Multiple orthogonal polynomials*, J. Comput. Appl. Math. **99** (1998) 423–447.

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[8] A. Doliwa, *Non-commutative Hermite–Padé approximation and integrability*, Lett. Math. Phys. **112**, 68 (2022) (17 pp.).

[9] A. Doliwa, *Non-autonomous multidimensional Toda system and multiple interpolation problem*, J. Phys. A: Math. Theor. **55**, 505202 (2022) (17 pp.).

[10] A. Doliwa, *Determinantal approach to multiple orthogonal polynomials, and the corresponding integrable equations*, Stud. Appl. Math. **153** (2024) e12726 (26 pp.).

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- **Maxime Fairon - Representations of double quasi-Poisson algebras in types B, C, D**

  Abstract: Double (quasi-)Poisson brackets were introduced on associative algebras by Van den Bergh to induce a (quasi-)Poisson structure on their representation spaces naturally equipped with a GL-action (type A). If there exists a compatible involutive anti-automorphism on the underlying associative algebras, Olshanski and Safonkin proved that this construction generalizes for double Poisson brackets on twisted representation spaces (types B, C, D). The main new result is the analogous generalization over a semisimple base, and for double quasi-Poisson brackets. To illustate this theory, I will explain how to recover the Poisson structure of character varieties for the orthogonal and symplectic groups. I will also introduce a definition of (multiplicative) quiver varieties outside the type A case. The talk will end with an outlook towards an application to classical integrable systems. This is based on an ongoing work with Semeon Arthamonov (BIMSA).

- **Laszlo Feher - Integrable systems from Poisson reductions of generalized Hamiltonian torus actions**

  Abstract: First, we develop sufficient conditions for guaranteeing that an integrable system with symmetry group K on a manifold M descends to an integrable system on a dense open subspace of the quotient Poisson space M/K and on its symplectic leaves. Then, we present applications to reductions of master systems on cotangent bundles and Heisenberg doubles of compact Lie groups and to integrable systems on moduli spaces of flat connections. In almost all examples, the term 'integrability' refers to degenerate integrability, alias superintegrability. The talk is based on a forthcoming joint paper with Maxime Fairon.

- **Guilherme Feitosa de Almeida - On the correspondence between statistical Manifolds and flat F-Manifolds in hyperbolic geometry**

  Abstract: Dubrovin-Frobenius manifolds provide a unifying framework connecting diverse fields, such as theoretical physics, algebraic geometry, and differential equations, offering new ways to understand and solve challenges across these areas. These manifolds naturally arise in parameter spaces of statistical theories, linking different disciplines and offering insights into phenomena ranging from wave dynamics to the structure of parameterized systems. Building on this foundation, I will present my recent work on the correspondence between statistical manifolds and Flat F-manifolds in hyperbolic geometry. Statistical manifolds, which are central to data analysis and machine learning, offer a mathematical framework for modeling probabilistic systems and biological dynamics. Flat F-manifolds, in contrast, capture relationships in integrable systems and algebraic structures. This correspondence suggests a geometric bridge that enriches both domains and opens new avenues for interdisciplinary exploration.

- **Evgeny Ferapontov - Integrable Lagrangians, modular forms and degenerations**

  Abstract: I will discuss integrable Lagrangians whose Lagrangian density f depends on first-order partial derivatives of a function u of three independent variables.

  It was known that the integrability conditions constitute an involutive system S for the Lagrangian density f whose solution space is 20-dimensional, supplied with a locally free action of a 20-dimensional symmetry group G. Furthermore, the 'generic' integrable density f has remarkable modular properties.

In this talk I will discuss non-generic densities which correspond to G-orbits of lower dimensions. The approach is based on the observation that every non-generic density must have a nontrivial stabiliser under the action of G, that is, every non-generic density is an invariant solution of system S. This allows one to apply the machinery of symmetry analysis of differential equations.

Based on joint projects with S. Opanasenko and F. Clery, A. Odesskii, D. Zagier.

- **Claudia Fevola - KP solitons: tropical curves meet Grassmannians**

  Abstract: The KP equation, a partial differential equation describing nonlinear wave motion, has solutions linked to algebraic curves. Solitons, a special class of solutions, arise from rational nodal curves. Kodama and Williams explored real regular solitons and their connection to totally positive Grassmannians. Building on Abenda and Grinevich's work, I will discuss the relationship between real regular solitons, dual graphs of singular curves, Le-graphs, and cells in the totally positive Grassmannian.

- **Vladimir Gerdjikov - Generalized Fourier transforms for MNLS with constant boundary conditions**

  Abstract: The present paper extends the fundamental idea of AKNS to the multi-component nonlinear Schrödinger equations (MNLS) related to the symmetric spaces of C.I type with constant boundary conditions (CBC). We use generic asymptotics for the potential $Q \to Q_\pm$ for $x \to \pm\infty$, so that the asymptotic operators $L_\pm$ have 4 different eigenvalues and the relevant MNLS equation has two chemical potentials: ρ_1 and $\rho_2 < \rho_1$. As a consequence the continuous spectrum of L has varying multiplicity equal to: (a) 4 for $\lambda^2 \geq \rho_1^2$; (b) 2 for $\rho_1^2 \geq \lambda^2 \geq \rho_2^2$; and (c) $0 \leq \lambda^2 \leq \rho_2^2$. It is well known that MNLS related to $sp(4)$ are used as models of spin-1 Bose-Einstein condensate (BEC). We solve the direct and inverse scattering problems for the corresponding Lax operator L.

  Next, we use the fundamental analytic solutions of L to determine the minimal sets of scattering data of L. A generalization of the Wronskian relations allows us to construct a set of squared solutions of L, which allow one to map: (a) the x-derivative Q_x of the potential of L to the minimal set of scattering data and (b) the variations δQ of the potential to the variations of the minimal set of scattering data.

  Then we introduce the Green functions of the squared solutions and applying the contour integration method, prove the completeness relations for the squared solutions. Next, we expand $\mathrm{ad}_J^{-1} Q_x$ and $\mathrm{ad}_J^{-1} \delta Q$ over the squared solutions. Considering special type of variations due to the evolution, we obtain also the expansion of $\mathrm{ad}_J^{-1} Q_t$. Thus we demonstrate that the minimal sets of scattering data and their t-derivatives provide the expansion coefficients $\mathrm{ad}_J^{-1} Q_x$ and $\mathrm{ad}_J^{-1} Q_t$ over the squared solutions. We also introduce the recursion operator Λ, for which the squared solutions are eigenfunction. Thus we are able to describe the hierarchy of MNLS with CBC.

- **Claire Gilson - Soliton gases and ultra-discrete integrable systems**

  Abstract: In this talk I will consider ultra-discrete systems in the context of soliton gases. I will compare the rarefied gas where, due to the low density of particles, interaction are relatively rare, with the case of a dense gas where multiple interactions may happen simultaneously.

- **Georgi Grahovski - On the N-wave hierarchy with constant boundary conditions**

Abstract: In this talk, we will present the direct scattering transform for the N-wave resonant interaction equations with non-vanishing boundary conditions. For special choices of the boundary values $Q_{\pm}$, we outline the spectral properties of L, the direct scattering transform and construct its fundamental analytic solutions. Then, we generalise Wronskian relations for the case of constant boundary conditions.

Finally, using the Wronskian relations we derive the dispersion laws for the N-wave hierarchy and describe the NLEE related to the given Lax operator. The results are illustrated by an example of 4-wave resonant interaction system related to the algebra $sp(4, \mathbb{C})$.

Based on joint work with Vladimir S. Gerdjikov [1, 2].

[1] V. S. Gerdjikov, G. G. Grahovski, *On the* 3*-wave equations with constant boundary conditions*, PLISKA Stud. Math. Bulgar. **21** (2012), 217–236 [E-print: arXiv:1204.5346].

[2] V. S. Gerdjikov, G. G. Grahovski, *On the N-waves hierarchy with constant boundary conditions. Spectral properties*, Int. J. of Geom. Methods in Modern Physics (2024), to appear, [E-print: arXiv:2403.12925].

- **Derek Harland - Geometry of Lagrangian one-forms**

  Abstract: Lagrangian multiform theory is a variational framework for integrable systems. In this talk I will present a new formulation of Lagrangian one-forms that is rooted in differential geometry. Arguably, this new formulation is simpler and more powerful than the traditional formulation. I will describe an application to Hitchin's integrable system, where the Lagrangian one-form becomes a BF Lagrangian.

- **Pavlos Kassotakis - Non-Abelian elastic collisions and discrete analytic functions**

  Abstract: We extend the equations of motion that describe non-relativistic elastic collision of two particles in one dimension to an arbitrary associative algebra. Relativistic elastic collision equations turn out to be a particular case of these generic equations. Furthermore, we show that these equations can be reinterpreted as difference systems defined on the $\mathbb{Z}^2$ graph and this reinterpretation relates the linear and the non-linear approach of discrete analytic functions.

- **Rob Klabbers - Freezing elliptic spin Ruijsenaars models**

  Abstract: There are at least two seemingly distinct realms of quantum integrability. The first domain is formed by the (short-range) Heisenberg spin chains, connected to the quantum inverse scattering method, which play a role in many different contexts both in physics and mathematics. The second domain is formed by the Calogero-Sutherland (CS) models and their deformations, which are families of Schrödinger or difference operators with close ties to harmonic analysis, orthogonal Jack and Macdonald polynomials, and Knizhnik-Zamolodchikov equations. Their integrability follows from a connection to affine Hecke algebras.

- **Christian Korff - Ind-cluster algebras and the Sato-Segal-Wilson Grassmannian**

  Abstract: There is a bijection between solutions of the Kadomtsev-Petiashvili (KP) hierarchy and points on an infinite Grassmannian, now often simply referred to as "the Sato Grassmannian". This connection is made via the Pl"ucker coordinates, the expansion coefficients of the KP tau-function in the basis of Schur functions. That is, the KP tau-functions satisfy the Pl"ucker relations of all finite Grassmannians and their union forms what is known as the KP hierarchy. In other words, one considers the inductive limit of the coordinate rings of finite Grassmannians. A celebrated result is that the coordinate rings of finite Grassmannians carry a cluster algebra structure. We show that this cluster algebra structure can be extended to the inductive limit by introducing ind-cluster algebras.

This is ongoing joint work with Sira Gratz, Aarhus Universitet.

- **Irfan Mahmood - Quantum Painlevé second Lax pair and quantum (matrix) analogues of classical Painlevé II equation**

  Abstract: This talk encloses discussion on quantum Painlevé second Lax pair that explicitly involves the Planck constant $\hbar$ and an arbitrary field variable v which distinguish it from the existing Flaschka–Newell Painlevé second Lax pair. We show that Flaschka–Newell pair appears as a special case of our quantum Painlevé second Lax pair. It is shown that the compatibility of quantum Painlevé second Lax pair simultaneously yields a quantum Painlevé equation and a quantum commutation relation between field variable v and independent variable z. We also show that the field variable v with different choices yields various analogs of classical Painlevé second equation as matrix Painlevé second equation, derivative matrix Painlevé second equation, and noncommutative Painlevé second equation. Further, we construct the gauge equivalence of quantum Painlevé second Lax pair whose compatibility condition generates a quantum P34 equation involving $\hbar$ with power $+1$ that brings the classical P34 equation close to its quantum analogs as compared to the existing one which carries $\hbar$ with power $+2$.

- **Maciej Nieszporski - Integrable discretizations of nonlinear sigma models (chiral models)**

  Abstract: I will begin with a review of known results concerning the problem of such discretizations of nonlinear sigma models that exhibit integrable properties. Then I will present the latest achievements in this field. I will conclude the lecture with open problems in this area. I will also emphasize the role of the results, first, in the discretization of integrable equations of General Relativity, and second, in the field of discrete analytic functions.

- **Vladimir Novikov - Integrability of nonabelian differential–difference equations: the symmetry approach**

  Abstract: We extend the symmetry approach to tackle the integrability problem for evolutionary differential-difference equations on free associative algebras, also referred to as nonabelian differential-difference equations. This approach enables us to derive necessary integrability conditions, determine the integrability of a given equation, and make progress in the classification of integrable nonabelian differential-difference equations. This work involves establishing symbolic representations for the nonabelian difference algebra, difference operators, and formal series, as well as introducing a quasi-local extension for the algebra of formal series within the context of symbolic representations. Applying this formalism, we solve the classification problem of integrable skew-symmetric quasi-linear nonabelian equations of orders $(-1, 1)$, $(-2, 2)$, and $(-3, 3)$, consequently revealing some new equations in the process.

- **Casper Oelen - Elliptic automorphic Lie algebras and integrable systems**

Abstract: Automorphic Lie algebras are a class of infinite-dimensional Lie algebras over the complex numbers that naturally arise in integrable systems, in particular in the context of reduction of Lax pairs. They can be thought of as Lie algebras of meromorphic maps (usually with prescribed poles) from a compact Riemann surface X into a finite-dimensional Lie algebra $\mathfrak{g}$ which are equivariant with respect to a finite group G acting on X and on $\mathfrak{g}$, both by automorphisms. Independently of their origins in integrable systems, they show up in algebra as examples of equivariant map algebras.

We present a classification of automorphic Lie algebras in the case of $\mathfrak{g} = \mathfrak{sl}(2, \mathbb{C})$ and where X is a complex torus. The automorphic Lie algebras in this classification fall into two disjoint families of Lie algebras parametrised by the modular curve of $PSL(2, \mathbb{Z})$, except for four cases, all isomorphic to Onsager's algebra. Furthermore, we show that well-known algebras arising in integrable systems - such as Holod's hidden symmetry algebra of the Landau-Lifshitz equation and the Wahlquist-Estabrook prolongation algebra of the same equation - admit particularly simple descriptions deriving from the automorphicity perspective. They turn out to be isomorphic to a current algebra $\mathfrak{sl}(2, \mathbb{C}) \otimes R$, or to its direct sum with the two-dimensional abelian Lie algebra $\mathbb{C}^2$, in the latter case, where R is a suitable ring of elliptic functions invariant under the dihedral group D_2 of order 4. This talk is based on joint work with Sara Lombardo and Vincent Knibbeler.

- **Stanislav Opanasenko - Bi-Hamiltonian structure of WDVV equations**

  Abstract: It is known that in low dimensions WDVV equations are equivalent to a set of bi-Hamiltonian hydrodynamic-type systems. In the ongoing work with Raffaele Vitolo, we generalise these results to an arbitrary dimension.

- **Edoardo Peroni - Darboux transformations of non-commutative DNLS equations. A factorisation property**

  Abstract: The non-commutative derivative nonlinear Schrödinger equations (DNLS) were identified by Olver and Sokolov [1]. They consist of seven integrable PDEs with Lax representations provided by Tsuchida and Wadati [2]. In this talk we introduce the associated linear and quadratic Darboux transformations and matrices. We examine the factorisation of a polynomial Darboux matrix and we present a necessary condition based on quasi-determinants for when a degree n polynomial Darboux matrix can be decomposed into a linear Darboux matrix and one of degree $n-1$. We conclude with an application to DNLS equations. The deduced integrable non-commutative differential-difference equations (DΔEs) involve non-commuting constants, generalising known integrable lattices, such as Volterra, Ablowitz-Ladik, Merola-Ragnisco-Tu and relativistic Toda.

  [1] P. J. Olver and V. V. Sokolov, *Non-abelian integrable systems of the derivative nonlinear Schrödinger type.* Inverse Problems, 14(6):L5, 1998.

  [2] T. Tsuchida and M. Wadati, *Complete integrability of derivative nonlinear Schrödinger-type equations.* Inverse Problems, 15(5):1363, 1999.

- **Sergei Turytsin - From integrable equations to fibre-optic communication and lasers**

  Public lecture

- **Pol Vanhaecke - A discrete algebraically integrable recursion relation**

  Abstract: In a recent collaboration with Andy Hone and John Roberts, we studied a 4D birational map that appeared in a recent classification by Gubbiotti et al. of a specific type of 4D birational maps having two invariants. The main result is that this map is discrete algebraically integrable in a precise sense which I will explain in detail.

- **Mikhail Vasilev - Generalised spin Calogero-Moser systems from Cherednik algebras**

  Abstract: Integrable spin Calogero–Moser type systems with non-symmetric configurations of the singularities of the potential appeared in the work of Chalykh, Goncharenko, and Veselov in 1999. In this talk I will show how these examples as well as various generalisations could be derived by making use of the representation theory of Cherednik algebras. This is a joint work with Misha Feigin and Martin Vrabec.

- **Mats Vermeeren - Lagrangian multiforms and conservation laws**

  Abstract: Lagrangian multiform theory is a variational principle for hierarchies of commuting differential equations. Lagrangian 1-forms typically describe systems of commuting ODEs, Lagrangian 2-forms typically describe hierarchies of (1+1)-dimensional PDEs, and so on.

  In this talk, we give a number of examples of Lagrangian multiforms that break this pattern. We will present Lagrangian 2-forms that can be interpreted as conservation laws for 3-dimensional PDEs. This leads to a rich set of equations satisfied by those solutions to the 3-dimensional PDE for which the characteristic of the conservation law vanishes.

- **Jing Ping Wang - Algebraic quantisation of dynamical systems**

  Abstract: In this talk, we'll discuss a recently emerged approach to the problem of quantisation. In this algebraic approach, we switch the focus from deformations of Poisson manifolds to dynamical systems themselves and reformulate the problem of quantisation in terms of quantisation ideals. We will present examples of multi-quantum structures, which reproduce multi-Hamiltonian structures in the classical limits, and examples of non-deformation quantisations of dynamical systems when the algebra defined by the commutation relations remains noncommutative for any choice of the quantum (Planck-type) constant, which will include the results for Volterra chain, Toda lattice and Ablowitz-Ladik lattice. This is a joint work with S. Carpentier and A.V. Mikhailov.

- **Pavlos Xenitidis - Noncommutative discrete KdV equations, their symmetries and reductions**

  Abstract: Starting with the discrete noncommutative Hirota's Korteweg–de Vries (KdV) and the potential KdV equations and their Lax pairs, we derive differential-difference equations consistent with the KdV equations which may be viewed as generalised symmetries of the latter. We show that these differential-difference equations are related to a noncommutative modified Volterra equation via Miura transformations and demonstrate how they can be used for the reduction of the potential KdV equation to a noncommutative discrete Painlevé equation and a system of partial differential equations generalising the Ernst equation and the Neugebauer–Kramer involution. A Darboux and an auto-Bäcklund transformation for the Hirota KdV are presented and their relation to the noncommutative Yang-Baxter map H_{III}^{B} is given.